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Are gross margins of structured products priced in a market-consistent way? Evidence from the new issuer estimated value

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ARE GROSS MARGINS OF STRUCTURED PRODUCTS PRICED IN A MARKET-CONSISTENT WAY? EVIDENCE FROM THE NEW ISSUER ESTIMATED VALUE

JANIS BAUER^{a,1}, HOLGER FINK^{b,2}, AND EVA STOLLER^{b,3}

ABSTRACT

In May 2014, as part of an industry-wide transparency initiative, the issuing institutions of retail structured products in Germany established the so-called *issuer estimated value (IEV)*, a fair value for each product which is targeted to reflect the market price among professional traders. By reporting the IEV in the product information sheet, issuers implicitly make a statement on their gross margin which contains the bank's profits but also distribution and selling costs. The present paper provides a first empirical analysis on the adequacy of the reported IEVs and the corresponding gross margins. Restricting ourselves to two of the main product subclasses, discounter and capped bonus certificates on the DAX index, we can verify the disclosed gross margins using the popular stochastic volatility model of Heston, the Practitioner Black-Scholes model and the Nadaraya-Watson approach. Furthermore, we find that even though the gross margins have become transparent to investors, there is still a notable heterogeneity across the issuing banks.

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JEL SUBJECT CLASSIFICATION

C52, G10, G13

Retail structured products provide a straightforward and easy access for retail clients, wealth managers and institutional investors to all kind of desirable payoff functions without the necessity of margin accounts and access to the derivatives exchanges. In principle, a structured product is a package of derivatives that is issued by a financial institution as bearer bond. In Germany alone, which is one of the largest markets for structured products in the world, the current outstanding volume in such structured products amounts to EUR 68.8 billion¹. If one has decided for an investment, trading is either possible OTC, i.e. directly with the issuer via, e.g., an online broker or through an retail exchange like EUWAX or Börse Frankfurt. In most cases, the provider of liquidity is the issuing institution itself which usually commits to sell and buy back until maturity. However, as shorting is prohibited, this potentially offers a significant profit opportunity for the issuer.

While early academic papers (cf. Chen and Kensinger [1990] or Chen and Sears [1990]) focus on the US market, there exists also a short but meaningful list of papers considering the German one. In the following, we shall concentrate on studies on the more conservative investment certificates, leaving high risk leverage products like warrants or barrier options out (cf. Muck [2006], Wilkens and Stoimenov [2007], Schmitz and Weber [2012], Entrop et al. [2013], Baule and Blonski [2015], Fink et al. [2016] and Fink and Mittnik [2016] on these).

¹April 2016, Deutscher Derivate Verband [2016]

Model price of derivative components

- *Finance income*
- + *Expected hedging costs*

Issuer estimated value (IEV)

- + *Expected issuer margin**
- + *Distribution and selling costs*

Structured product price

- + *Front end load fee, where applicable*

Acquisition price for the investor

Exhibit 1: Structured product price components according to Deutscher Derivate Verband [2015]. The star indicates that beyond the issuer's expected profits, the expected issuer margin covers operational costs incurred by product structuring, admission, market making and settlement.

The first extensive empirical analysis was carried out by Wilkens et al. [2003] who investigate the pricing of 906 structured products on DAX and NEMAX All-Share single stocks traded in November 2001. The considered products are discount certificates and reverse convertibles for which, after estimating the dividend yield on each stock, a nearly perfect replication with suitable EUREX options is possible. Including an approximation for credit risk inherent in structured products using bond indices, their study estimates that reverse convertibles on DAX (NEMAX All-Share) stocks are priced on average 3.04% (3.89%) above their theoretical values, while discounters show an even larger margin of 4.2% (10.04%). Besides these results, Wilkens et al. [2003] find evidence for their order flow hypothesis, which postulates that the observed overpricing diminishes when the product approaches maturity.

In line with Wilkens et al. [2003], Stoimenov and Wilkens [2005] find support for their life-cycle hypothesis (which is similar to the order flow hypothesis) by analyzing 2,566 equity-linked structured products on the DAX and its constituents available on October 10th, 2002. To price the embedded derivatives, the authors apply the standard option pricing model of Black and Scholes [1973] for European plain vanilla and the closed-form formulas of Rubinstein and Reiner [1991] for barrier options. Their results indicate that the prices of DAX structured products lie on average 2.13% above their model-implied value at issuance, while they are 0.11% lower in the secondary market. To find indication for declining margins as maturity approaches, Stoimenov and Wilkens [2005] regressed the relative price deviations on the products' remaining lifetimes proving statistical significance.

Later on, Baule et al. [2008] were the first who addressed the influence of each individual issuer's credit risk by comparing the results of three different models for valuing discount certificates – the standard default-free Black and Scholes [1973] setup, the Hull and White [1995] framework, which assumes independence between market and credit risk, and a structural model which allows for correlation effects. It turns out that the priced-in margins calculated with the Hull and White [1995] framework are strictly larger than the those resulting from the structural model. Assuming the latter to be the more realistic setup, Baule et al. [2008] comes to the conclusion that Deutsche Bank prices with the lowest gross margins (0.67%), followed by UBS (0.84%), Commerzbank (0.91%), BNP Paribas (1.29%) and Société Générale (2.27%).

In a more current study, Baule [2011] analyzes the order flow hypothesis for a sample of 4,451 discount certificates on the DAX issued by 11 different institutions between November 2006 and December 2007. Using a Black-Scholes-type implied volatility model adjusted for credit risk, the author shows that margins have become comparably small with an overall average of 0.42% and a mean lifetime of 1.14 years. As an explanation for these seemingly surprising observations compared to earlier findings, Baule [2011] refers to an increased trading volume and a more competitive market. For the more risky path-dependent bonus certificates, Baule and Tallau [2011] estimate average issuer margins between 1.98% p.a. and 3.50% p.a. by applying the stochastic volatility model of Heston [1993] incorporating credit risk via the independence approach of Hull and White [1995].

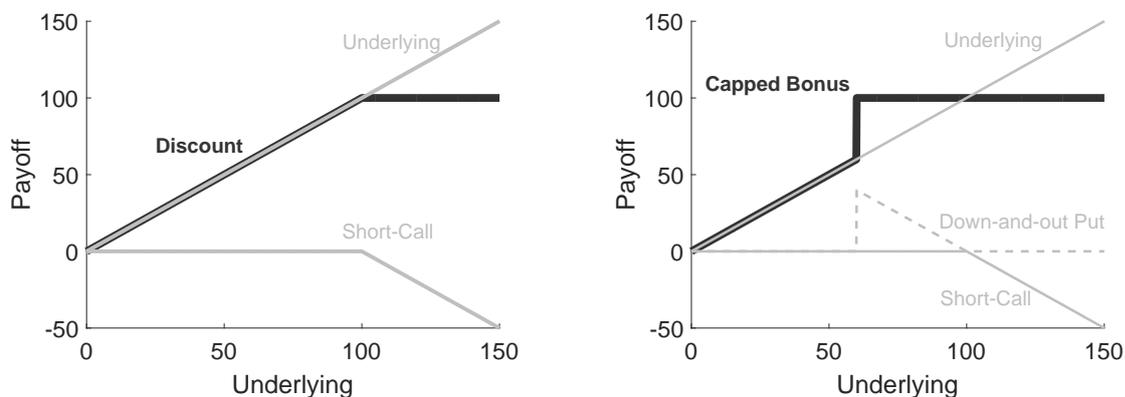


Exhibit 2: Illustration of discount and capped bonus certificate payoffs with a cap at 100 and a barrier at 60.

The most recent study² of Döhner et al. [2013] considers a selected representative and a random sample of 200 products each across all payoff profiles, deriving a mean issuer margin of 0.36% p.a. and 0.46% p.a., respectively. In addition to credit risk, they specifically include a flat barrier shift of 2% for bonus certificates and similar structured products which explains why they arrive at lower margins than, for example, Baule and Tallau [2011].

In May 2014, resulting from intensive discussions with regulators and the will to offer more transparency for retail clients, the Deutscher Derivate Verband (DDV) and the issuers in Germany established the so-called *issuer estimated value (IEV)*. The IEV is published in the product information sheet and should be determined by a market-consistent model price which is further adjusted for the finance income as the investor acts as a capital provider for the issuer. Additionally, the IEV also accounts for hedging costs which the issuing banks are facing. Exhibit 1 summarizes the definition of the IEV as outlined by the DDV. From there on, the issuers voluntarily commit to report the IEV of each investment certificate in its respective product information sheet which contains all relevant product features and a scenario analysis, cf. Deutscher Derivate Verband [2015]. When issuers disclose the IEV at the emission date, they also make an implicit statement on their expected gross margin, i.e. a value which contains the bank's profits but also potential distribution and selling costs.

Since then, to the best of our knowledge, no study has yet analyzed the adequacy of the reported IEVs and corresponding gross margins – a gap which we would like to start closing within this paper. In particular, we focus on the above already mentioned discount and capped bonus certificates which account in Germany for about 11.7% of total outstanding volume and 29.7% of the total monthly trading volume³. The respective payoff sketches are presented in Exhibit 2.

To reduce pricing uncertainty arising from different approaches of estimating dividend yields or illiquid hedging instruments, we focus on the DAX performance index as the underlying security. In particular, we compare the gross margins derived from IEVs of 714 structured products issued between September 30th and October 31st, 2015. In line with the recent studies of Baule [2011], Baule and Tallau [2011] and Döhner et al. [2013], we rely on two Black-Scholes-type pricing approaches using implied volatility surfaces and the stochastic volatility framework of Heston [1993]. Default risk will be incorporated by the Hull and White [1995] approach, but we will apply an additional haircut on the individual issuer's CDS accounting for credit/equity correlation. To account for hedging costs, we include a flat barrier shift and a volatility charge.

The present paper provides as a first empirical analysis on the adequacy of reported issuer estimated values and respective gross margins. A presumably broader study embedded in a larger research project, which we recently became aware of, seems currently to be conducted by Baule et al. [2016].

²The study of Döhner et al. [2013] was supported by the *Deutscher Derivate Verband (DDV)*, the issuers' lobby group in Germany.

³June 2016, cf. Deutscher Derivate Verband [2016]

The remainder of the paper is organized as follows: The first section describes the selection procedure of discount and capped bonus certificates. Afterwards, we provide a descriptive analysis of the gross issuer margins resulting from reported IEVs in the product information sheets. The following section then introduces the applied pricing frameworks explaining in detail how parameters are calibrated and hedging costs applied, before we present the results of our pricing study in the subsequent section. The paper closes with a conclusion summarizing our main findings.

Data Selection

To empirically investigate the adequacy and fairness of reported IEVs, we focus on discount and capped bonus certificates which represent two major subclasses of retail structured products. In the following, we consider DAX index certificates issued between September 30th and October 30th, 2015, a time frame which is characterized by a steady and rising market combined with a falling volatility following the China-induced sell-off in the foregoing summer. For data collection, we use OnVista, a website which provides structured product selection tools and contains all exchange-listed products, leading to a total of 1504 discount and 3208 capped bonus certificates.

To ensure homogeneity of the certificates' characteristics and a liquid and stable reference market for hedging and pricing purposes, we consider only product lifetimes larger than 3 and smaller than 24 months. Now we filter by maturity and identify time frames in which at least 2 issuers are present with more than 5 single products which have a (nearly) similar counterpart from another issuer. Additionally, due to the larger set for capped bonus certificates, we focus there on the quarterly expiries which usually allow for a more liquid hedging market. For discounters, these time frames are June-August, October-November 2016, February-March and September 2017, while for capped bonus certificates March, June, December 2016 as well as March 2017 represent these periods. Now, we choose all products which have at least one (nearly) similar counterpart which finally results in a set of 501 discount and 259 bonus certificates, cf. Exhibit 3. As a result, the data set contains discounters issued by Commerzbank (144), HSBC (85), Citigroup (73), DZ Bank (62), Vontobel (55), Goldman Sachs (20) and BNP Paribas (17).

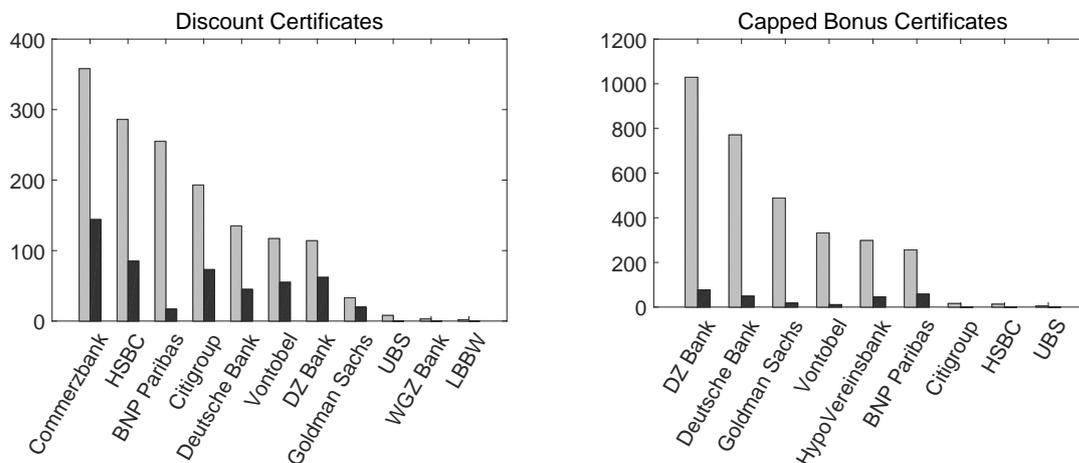


Exhibit 3: Data overview of our discount (left) and capped bonus certificates (right). Grey bars represent all available products while black bars indicate how much of these were chosen by the explained selection procedure.

In the subsample of capped bonus certificates we have DZ Bank (77), BNP Paribas (58), Deutsche Bank (49), Goldman Sachs (18) and Vontobel (11). A detailed list of product identifiers can be found in the appendix. Note that the 46 capped bonus certificates of HypoVereinsbank (HVB) shown in Exhibit 3 have to be dropped from the final data set as HVB did not provide any product information sheets during October 2015. Finally, for all 714 structured products, closing bid and ask quotes at 5:30 pm Frankfurt time are obtained from the EUWAX exchange while IEVs, issue prices and dates are extracted out of the product information sheets downloaded from each issuer's webpage.

Issuer	# Products	Reported gross margin in % p.a.				Time-to-maturity in years			
		Mean	Std.	Min.	Max.	Mean	Std.	Min.	Max.
<i>Panel A: Discount certificates</i>									
BNP Paribas	17	1.41	0.10	1.34	1.66	1.20	0.22	0.77	1.38
Citigroup	73	0.68	0.12	0.31	0.82	1.12	0.14	0.73	1.44
Commerzbank	144	0.05	0.01	0.04	0.08	1.31	0.17	1.12	1.45
Deutsche Bank	45	0.11	0.23	0.02	0.81	1.38	0.22	0.69	1.46
DZ Bank	62	0.01	0.00	0.00	0.02	1.57	0.48	1.02	2.02
Goldman Sachs	20	0.73	0.03	0.69	0.76	0.78	0.22	0.67	1.45
HSBC	85	0.18	0.04	0.04	0.27	1.20	0.40	0.72	1.99
Vontobel	55	1.58	0.31	1.38	2.20	1.62	0.46	0.83	1.98
<i>Overall</i>	501	0.40	0.54	0.00	2.20	1.31	0.37	0.67	2.02
<i>Panel B: Capped bonus certificates</i>									
BNP Paribas	58	2.73	0.52	2.28	3.51	0.88	0.35	0.42	1.19
Deutsche Bank	49	0.42	0.20	0.02	0.68	0.81	0.35	0.46	1.22
DZ Bank	77	0.20	0.01	0.20	0.22	0.92	0.35	0.39	1.49
Goldman Sachs	18	1.28	0.02	1.27	1.31	1.16	0.36	0.45	1.47
Vontobel	11	1.47	0.08	1.39	1.57	1.01	0.54	0.39	1.48
<i>Overall</i>	213	1.10	1.10	0.02	3.51	0.91	0.37	0.39	1.49

Exhibit 4: Descriptive statistics of annualized reported gross margins (in %) for discount and capped bonus certificates.

Descriptive Statistics

In the following section, we take reported IEVs from the certificates' product information sheets and set them into a relation to the issue price to obtain an implicitly reported gross margin for each product. To obtain a more balanced panel, we further divide by the time to maturity T to obtain an annualized reported gross margin which is specifically defined by,

$$\text{Reported gross margin p.a.}_i = \frac{1}{T_i} \left(1 - \frac{\text{IEV}_i}{\text{Issue Price}_i} \right). \quad (1)$$

Exhibit 4 provides summary statistics of the reported annualized gross margins for the overall data set ($n = 714$). Throughout this paper, we consider annualized gross margins and the only exceptions to this procedure are Exhibit 5 and Exhibit 6 where maturity-dependencies are illustrated. In total, the reported gross margins show an overall average of 0.61% p.a. and an average time to maturity of 1.19 years.

Discount Certificates

In our data sample, discount certificates show a mean reported gross margin at emission of 0.40% p.a. for an average time to maturity of 1.31 years. These numbers are only slightly lower than the average margin in the analysis of Baule [2011] who derives 0.42% with an average time to maturity of 1.14 years for discount certificates on the DAX between November 2006 and December 2007. Also, Döhrer et al. [2013] calculate an average margin of 0.39% p.a. at emission with an average time to maturity of 2.01 years for 200 discount certificates. All these findings are in line with the industry-wide belief that the banks' profit made has declined over the last years due to increasing competition.

Although, as any comparison across banks should be treated with care, one can still deduce that Vontobel seems to offer the discounters with the highest gross margins ranging from 1.38% p.a. up to 2.20% p.a. In contrast to the findings of Baule [2011], DZ Bank reports by far the lowest margin with at most 0.02% p.a. of the product price with an average time to maturity of 1.57 years. However, this looks more like a technical error in their calculations – a conjecture which

is further strengthened when considering the scatter plot in the third row of Exhibit 5. In the study of Baule [2011], DZ Bank is shown to price with an average margin of more than 1% for an average time to maturity of about 1.08 years.

The reported gross issuer margins are illustrated in detail in Exhibit 5. Considering the second line of Exhibit 5, reported gross margins of Commerzbank's discounters interestingly seem comparably low and are decreasing in cap and increasing in time to maturity. In contrast, HSBC's structured product show margins which are increasing in cap and increasing in time to maturity. At last, Citigroup reports 10-times higher reported gross margins than Commerzbank which even show some kind of inverted smile. The reported gross margins of Vontobel's discount certificates are constant in cap and in general increasing in time to maturity.

However, this effect is not consistent for all lifetimes as products with a time to maturity of 16 months exhibit higher margins than structured products with the same cap and a time to maturity of 23 months. The discounters of Deutsche Bank show a higher reported gross margin for products with a shorter lifetime (around 0.55% for a time to maturity of 8 months) than structured products with a considerably longer one (around 0.03% for a time to maturity of 17 months). For Goldman Sachs and BNP Paribas, we only have just a few discount certificates in the sample which show issuer margins increasing in time to maturity which is what we might usually expect.

Capped Bonus Certificates

The reported gross issuer margins for the subsample of 213 capped bonus certificates range from 0.02% p.a. up to 3.51% p.a. and result in an average of 1.10% p.a. for a mean time to maturity of 0.91 years. These findings are roughly in line with Döhrer et al. [2013] who calculate an expected issuer margin of 0.90% p.a. at emission for the 200 bonus certificates included in their data set. However, compared to Baule and Tallau [2011], gross margins seem distinctly lower which can be explained by the barrier shift which is usually applied by issuers to account for gap or jump risk.

Exhibit 6 provides the issuer-specific scatter plots of the implied margins as a function of the digital (-risk) for fixed time to maturity which is defined for the i -th capped bonus certificate by

$$\text{Digital}_i = \text{Bonus Level}_i - \text{Barrier Level}_i . \quad (2)$$

Since digital is the difference between the bonus level and the barrier, it represents that part of the payoff, that will be lost in case of a barrier event. However, one needs to interpret this characteristic with care as the absolute prices of capped bonus certificates explicitly depend on the bonus and barrier levels and not only on their difference.

The reported issuer margins of DZ Bank's capped bonus are constant in digital, increasing in maturity and range from 0.20% p.a. for short-term lifetimes up to 0.22% p.a. for times to maturity of about 18 months. Comparably, the respective structured products of BNP Paribas show constant margins in digital which still are increasing in lifetime, but tend between 2.28% p.a. and 3.51% p.a. Deutsche Bank is somehow special as we can clearly see that their products' reported gross margins are slightly decreasing in digital while ranging between 0.02% p.a. and 0.68% p.a. Finally, Goldman Sachs and Vontobel seem to report constant margins for all digitals, even though these findings have to be interpreted with care as both issuers are represented by less than 20 products in our data set.

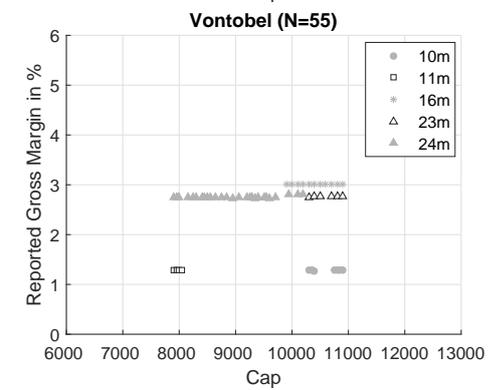
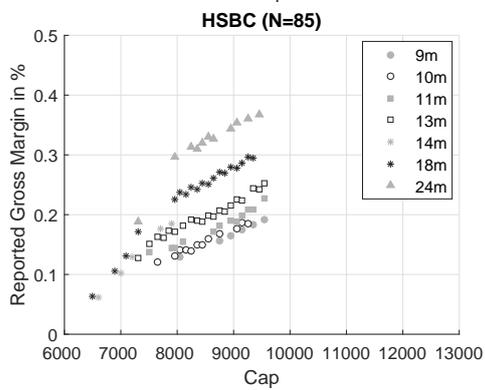
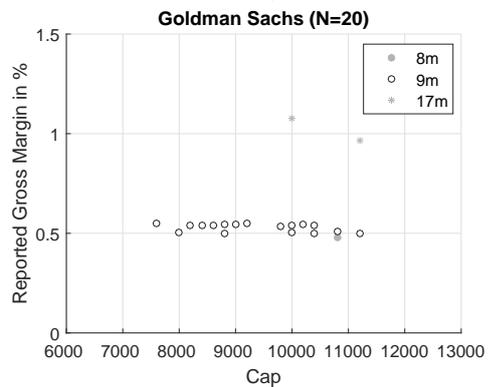
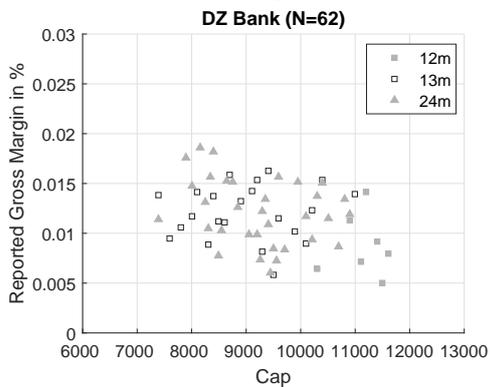
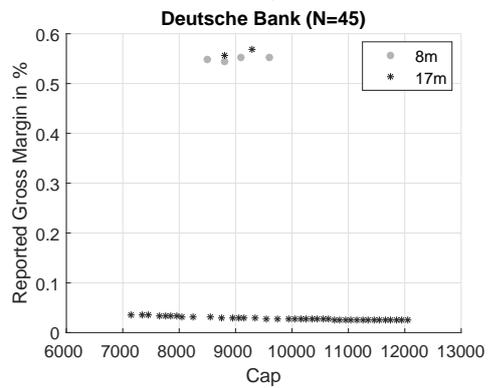
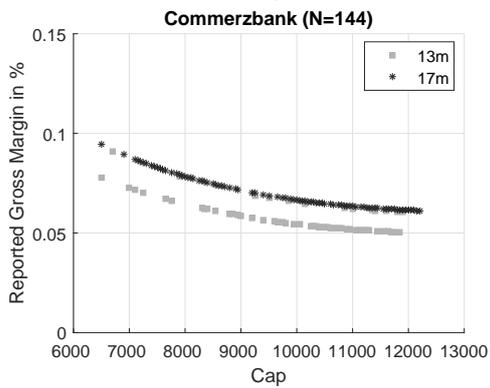
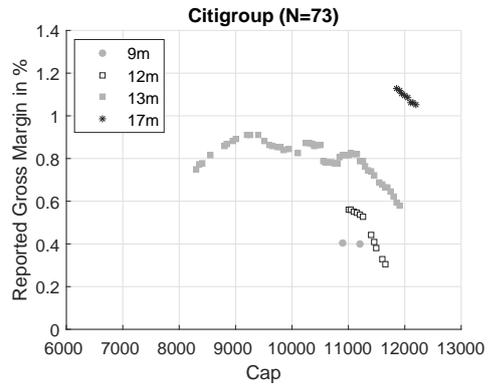
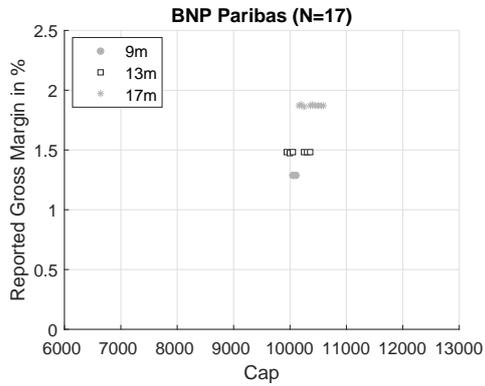


Exhibit 5: Reported gross margins of discount certificates (in %).

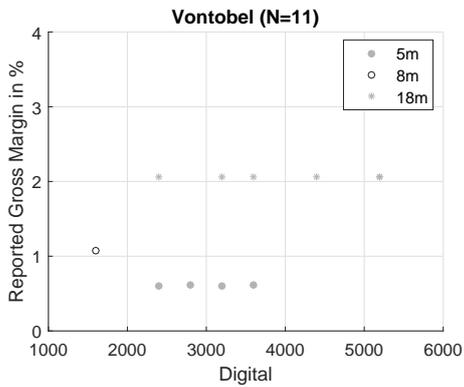
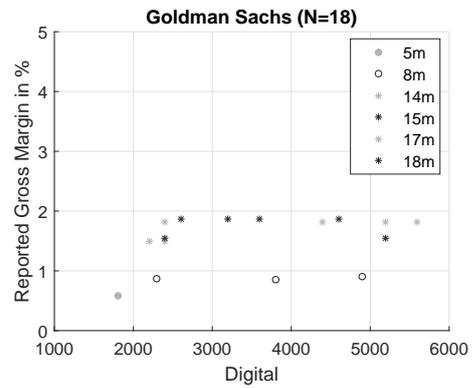
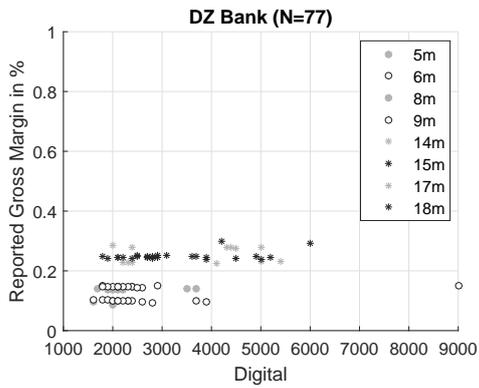
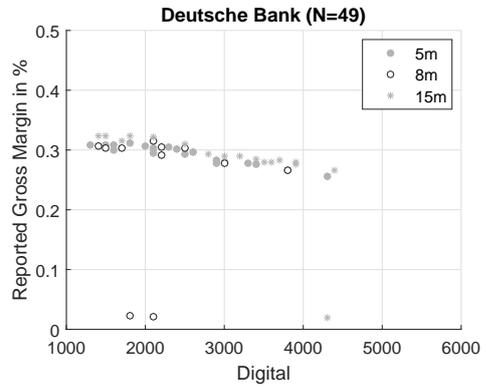
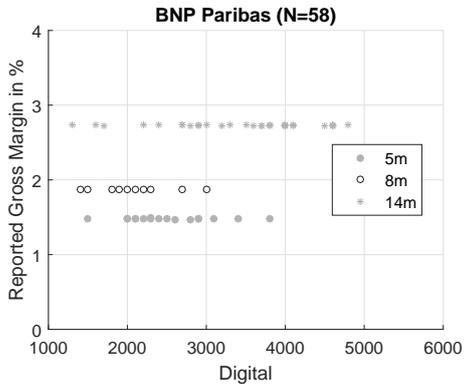


Exhibit 6: Reported gross margins of capped bonus certificates (in %).

Pricing Methodology

In this section, we explain our pricing methodology by recalling the option pricing models used in the empirical analysis, describing the calibration procedure and explaining the rationale behind model price adjustments that account for hedging costs and default risk.

Embedded Option Components

The derivative component of a discount certificate D_i can be replicated by a long position in a zero-strike call C_i^0 and a short position in a European call option C_i :

$$D_i = C_i^0 - C_i \quad (3)$$

For a capped bonus certificate, the replication scheme consists of a long position in a zero-strike call C_i^0 , a long position in a down-and-out put P_i^{DO} and a short European call C_i which features the cap level:

$$CB_i = C_i^0 + P_i^{DO} - C_i. \quad (4)$$

For valuation of these derivative components, possible choices for the option pricing models are manifold. To reduce uncertainty arising from the model choice, we implement three entirely different approaches, namely the commonly used Heston [1993] stochastic volatility model, the Dumas et al. [1998] local implied volatility model, which we refer to as the Practitioner Black-Scholes (PBS) model, and the non-parametric Nadaraya-Watson approach.

Heston Model

Regarding the Heston model, we assume that the underlying value and variance are determined by the following dynamics under the risk-neutral measure:

$$\frac{dS_t}{S_t} = r dt + \sqrt{v_t} dB_t^1, \quad S_0 \geq 0, \quad (5)$$

$$dv_t = \kappa(\theta - v_t) dt + \sigma\sqrt{v_t} dB_t^2, \quad v_0 \geq 0. \quad (6)$$

Here, S_t denotes the underlying asset value in t , r equals the continuously compounded risk-free rate and $\sqrt{v_t}$ denotes a stochastic volatility process. The time-varying variance v_t is driven by a Cox-Ingersoll-Ross (CIR) process with reversal speed rate $\kappa > 0$, long run variance $\theta \geq 0$ and the volatility of variance parameter $\sigma > 0$. The asset value and variance process is initialized with $S_0 \geq 0$ and $v_0 \geq 0$, respectively. Furthermore, $(B_t^1)_{t \geq 0}$ and $(B_t^2)_{t \geq 0}$ are two Brownian motions correlated with $\rho \in [-1, 1]$. For numerical robustness, we use the call price formula presented by Lewis [2001] in its most common variant which can be expressed as

$$C = S_0 - \frac{1}{\pi} \sqrt{S_0 K} e^{-rT/2} \int_0^\infty \operatorname{Re} \left[e^{i u m} \phi \left(u - \frac{i}{2} \right) \right] \frac{du}{u^2 + 0.25}. \quad (7)$$

In this expression, ϕ equals the characteristic function of log asset returns (see, for instance, Albrecher et al. [2007]), K denotes the strike price and $m = \log\left(\frac{S}{K}\right) + rT$ is a measure of option moneyness. In the following, we use a 100-points Gauss-Laguerre quadrature which ensures a high accuracy of the computed model price.

PBS Model

Dumas et al. [1998] present an implied volatility model based on deterministic volatility functions. In this approach, the Black-Scholes implied volatility σ_{BS} is directly determined by a polynomial of option characteristics. In particular, we use the moneyness and time-to-expiry of the respective option as explanatory variables:

$$\sigma_{BS}(M, T) = a_0 + a_1 M + a_2 M^2 + a_3 T + a_4 T^2 + a_5 TM + \varepsilon \quad (8)$$

Here, a is a vector of regression coefficients, T denotes the option's time-to-maturity, $M = \frac{S_0}{K}$ represents the option's simple moneyness and ε equals a standard normally distributed error term. The linear form of equation (8) gives the

opportunity to obtain closed form estimates for the coefficients using standard least squares methods. When regression coefficients are available, then options can be priced by plugging the respective option characteristics into equation (8) to obtain a volatility parameter σ_{BS} which then can be subsequently used in the standard Black-Scholes formulas.

Nadaraya-Watson

Both the Heston and PBS model are typically used to provide a global fit to discrete implied volatility observations. However, it is also common practice to use non-parametric methods, such a spline interpolation or kernel regressions which provide a local fit to the option data. Following Ait-Sahalia and Lo [1998] and Fengler [2006], we additionally use a non-parametric Nadaraya-Watson (NW) estimator to obtain a smooth volatility surface:

$$\sigma_{BS}(M,T) = \frac{\sum_{k=1}^{N_t} \sigma_{BS}(M_k, T_k) \phi(M - M_k, T - T_k)}{\sum_{k=1}^{N_t} \phi(M - M_k, T - T_k)} \quad (9)$$

where

$$\phi(x,y) = \frac{1}{2\pi} \exp\left(-\frac{x^2}{2h_x}\right) \exp\left(-\frac{y^2}{2h_y}\right) \quad (10)$$

is a bivariate Gaussian kernel. In this formula, T_k and $M_k = \frac{S_0}{K_k}$ denote the k -th option's time-to-maturity and simple moneyness. The bandwidth parameters h_x, h_y control the degree of smoothness and have to be specified separately. Using a cross-validation procedure, we determine the bandwidth for each day and maturity date to account for the different sparseness of data. Further, we determine the kernel bandwidth in the time dimension by the minimum distance to the next maturity date.

Pricing Down-and-Out Options

The valuation of the embedded path-dependent barrier option of a capped bonus certificate imposes more difficulties than the pricing of vanilla options. For the Heston model, we price the down-and-out put option using Monte Carlo simulations. To prevent numerical instabilities of the variance process near zero, we use the Andersen and Brotherton-Ratcliffe [2001] moment matched log-normal approximation scheme to simulate trajectories of the underlying. As the simulation is very time-consuming, we restrict the stock path simulation to 1-minute time intervals. The mean payoff resulting from 1,000,000 sample paths is discounted in order to obtain the fair value of the embedded down-and-out put,

$$P_i^{DO} = \exp(-rT_i) \left(\frac{1}{1,000,000} \sum_{s=1}^{1,000,000} (K_i - S_T^s)^+ \mathbb{I}_{\{\min_{t \in [0, T]} S_t^s > H_i\}} \right). \quad (11)$$

Additionally, we use the value of a simple put option and the drift rate as control variates for effective variance reduction of the Monte-Carlo estimate according to Boyle et al. [1997].

For the Black-Scholes model, there exists a closed-form solution of Rubinstein and Reiner [1991] for this option type. Their solution exploits the barrier option in-out parity, i.e. prices of a down-and-out put option can be expressed as a difference between the value of a plain vanilla put and the respective down-and-in option

$$P_i^{DO} = P_i - P_i^{DI}, \quad (12)$$

where

$$\begin{aligned} P_i^{DI} = & -S_0 \mathcal{N}(-x_1) + K_i \exp(-rT_i) \mathcal{N}\left(-x_1 + \sigma_{BS} \sqrt{T_i}\right) + S_0 \left(\frac{H_i}{S_0}\right)^{2\lambda} (\mathcal{N}(y) - \mathcal{N}(y_1)) \\ & - K_i \exp(-rT_i) \left(\frac{H_i}{S_0}\right)^{2\lambda-2} \left(\mathcal{N}\left(y - \sigma_{BS} \sqrt{T_i}\right) - \mathcal{N}\left(y_1 - \sigma_{BS} \sqrt{T_i}\right)\right) \end{aligned} \quad (13)$$

and

$$\begin{aligned} \lambda &= \frac{r + \frac{\sigma_{BS}^2}{2}}{\sigma_{BS}^2}, & y &= \frac{\log\left(\frac{H_i^2}{S_0 K_i}\right)}{\sigma_{BS} \sqrt{T_i}} + \lambda \sigma_{BS} \sqrt{T_i}, \\ x_1 &= \frac{\log\left(\frac{S_0}{H_i}\right)}{\sigma_{BS} \sqrt{T_i}} + \lambda \sigma_{BS} \sqrt{T_i}, & y_1 &= \frac{\log\left(\frac{H_i}{S_0}\right)}{\sigma_{BS} \sqrt{T_i}} + \lambda \sigma_{BS} \sqrt{T_i}. \end{aligned} \quad (14)$$

This formula assumes that the underlying is observed continuously and that the barrier H can be breached at any time. However, the DAX index is observed discretely every second. To include this small discrepancy in the calculation, Broadie et al. [1997] provided a continuity correction for the closed-form solution of a barrier option under the Black-Scholes model. When T/m represents the size of a time interval between observations, a corrected closed-form solution can be obtained by an adjustment of the barrier H to $H \cdot \exp(-0.05826 \sigma_{BS} \sqrt{T/m})$.

Default Risk and Hedging Costs

According to the IEV definition, the raw model price obtained by the sum of the individual option components has to be adjusted for hedging costs and finance income. Assuming independence of the bank's default risk and the underlying of the derivative, Hull and White [1995] proposed to discount the default-free model price with the issuer's credit

risk may lead to incorrect results. In particular, issuing banks usually apply a certain transformation / haircut to their respective CDS rates. From a practical perspective, this can be explained by the following rationale: For products that profit from rising markets (e.g. discount and capped bonus certificates), a dropping underlying usually leads to clients selling off their structures early. However, in such a downside market, the bank's funding costs reflected in their CDS rates rise. In total, when the issuer actually needs the additional funding provided by structured products, he might face larger buybacks – a risk he does not have when financing himself via bonds. Considering the relevance of correlation as illustrated by Baule et al. [2008], we reduce the credit spreads s_j by a flat haircut of 20 basis points. However, if one is interested in results without this adjustment, the 20 basis points p.a. can easily be added again to the derived annualized gross margins in the result section.

Also, hedging costs have to be reflected in the model-based fair value. Hedging a discount and a capped bonus certificate requires the issuer to short a call option at the EUREX exchange. By selling (and potentially re-buying) the option, the issuer has to pay the bid-ask spread. To account for these trading costs in the calculation of the model-based fair value, we consider the median bid-ask spread in terms of implied volatility of all call options traded in September 2015 and thus reduce our respective pricing volatilities for the short call parts by 0.62% per year. For capped bonus certificates, we additionally follow the approach of Döhner et al. [2013] and adjust the barrier with a flat barrier shift of 2% to account for gap risk. This means, the embedded down-and-out put option is evaluated at a 2% smaller barrier to reflect, for example, jump risk in the underlying.

The model-based fair values of a discounter and a capped bonus certificate of issuer j are thus defined as

$$FV_i^D = e^{-(s_j - 0.002)T_i} (C_i^0 - C_i^*) \quad \text{and} \quad FV_i^{CB} = e^{-(s_j - 0.002)T_i} (C_i^0 + P_i^{DO*} - C_i^*), \quad (15)$$

where C_i^* is the respective theoretical call price with reduced implied volatility and P_i^{DO*} is the value of the down-and-out put evaluated at a 2% lower barrier H^* , e.g. $H^* = 0.98 \cdot H$.

Model-Based Gross Margins

The resulting fair values are compared to the observed 5:30 pm ask prices of the certificates on their issue dates. Thus, the annualized gross issuer margin of a discount certificate is defined by

$$\text{Model-based gross margin p.a.}_i^D = \frac{1}{T_i} \left(1 - \frac{FV_i^D}{\text{Ask Price}_i^{\text{close}}} \right), \quad (16)$$

and the annualized gross margin for a capped bonus certificate via

$$\text{Model-based gross margin p.a.}_i^{CB} = \frac{1}{T_i} \left(1 - \frac{FV_i^{CB}}{\text{Ask Price}_i^{\text{close}}} \right), \quad (17)$$

respectively.

Model Calibration

We obtain daily closing prices at 5:30 pm of the DAX index and bid / ask quotes of European-style DAX index options from EUREX which can be accessed by Thomson Reuters Datastream. To begin with, we apply different exclusion criteria to obtain option quotes that are reliable and as close as possible to the actual trades. First, we drop all option quotes where either the bid or the ask price is not available or does not satisfy standard no-arbitrage conditions. Second, we only consider actively traded options with more than three months or less than two years time-to-expiration, a range which captures the minimum and maximum time-to-maturity of our certificates. Third, we drop in-the-money options and out-of-the-money options that do not satisfy the moneyness criterion $|S_0/K - 1| \leq 0.4\sqrt{T}$.

As the characteristics of the embedded option may differ from those traded on the market, we sequentially calibrate the Heston and PBS model at each emission date to the option data. For this purpose, we obtain Black-Scholes implied volatilities from mid prices $\mathcal{P}_{\text{Market}}$ of the remaining call and put options and match implied volatilities computed from the Heston or PBS model prices $\mathcal{P}_{\text{Model}}$. As an objective function, we choose model parameters in a way such that the root mean squared errors of implied volatility (IVRMSE) are minimized,

$$\text{IVRMSE} = \sqrt{\frac{1}{N_t} \sum_{k=1}^{N_t} \left(\sigma_{BS}(\mathcal{P}_{\text{Model},k}) - \sigma_{BS}(\mathcal{P}_{\text{Market},k}) \right)^2}, \quad (18)$$

where N_t denotes the number of options at day t . The Nadaraya-Watson approach is non-parametric and does not have to be calibrated.

To approximate the risk-free interest rate, we use maturity-matched Euro overnight swap rates which are available up to maturities of 10 years. The credit spreads are obtained from the senior debt one-year CDS of each issuer. As no CDS or bond yields are available for Vontobel, we make an approximation through an average of CDS spreads from UBS and Credit Swiss which have a similar Moody's long term issuer rating and operate in the same market.

Exhibit 7 shows Heston and PBS model parameters and the IVRMSE in-sample fit obtained from daily calibration during the period October 2015. The calibrated Heston and PBS model parameters are fairly stable over time, except the spot variance parameter decreased noticeably after the first days. A less pronounced decrease can also be observed for the PBS intercept a_0 , however the spot variance is also affected by the moneyness coefficients a_1 and a_2 . The average in-sample fit of the PBS model is with 0.0041 slightly better than for the Heston model with 0.0050 which can be explained by the increased flexibility due to the additional model parameter.

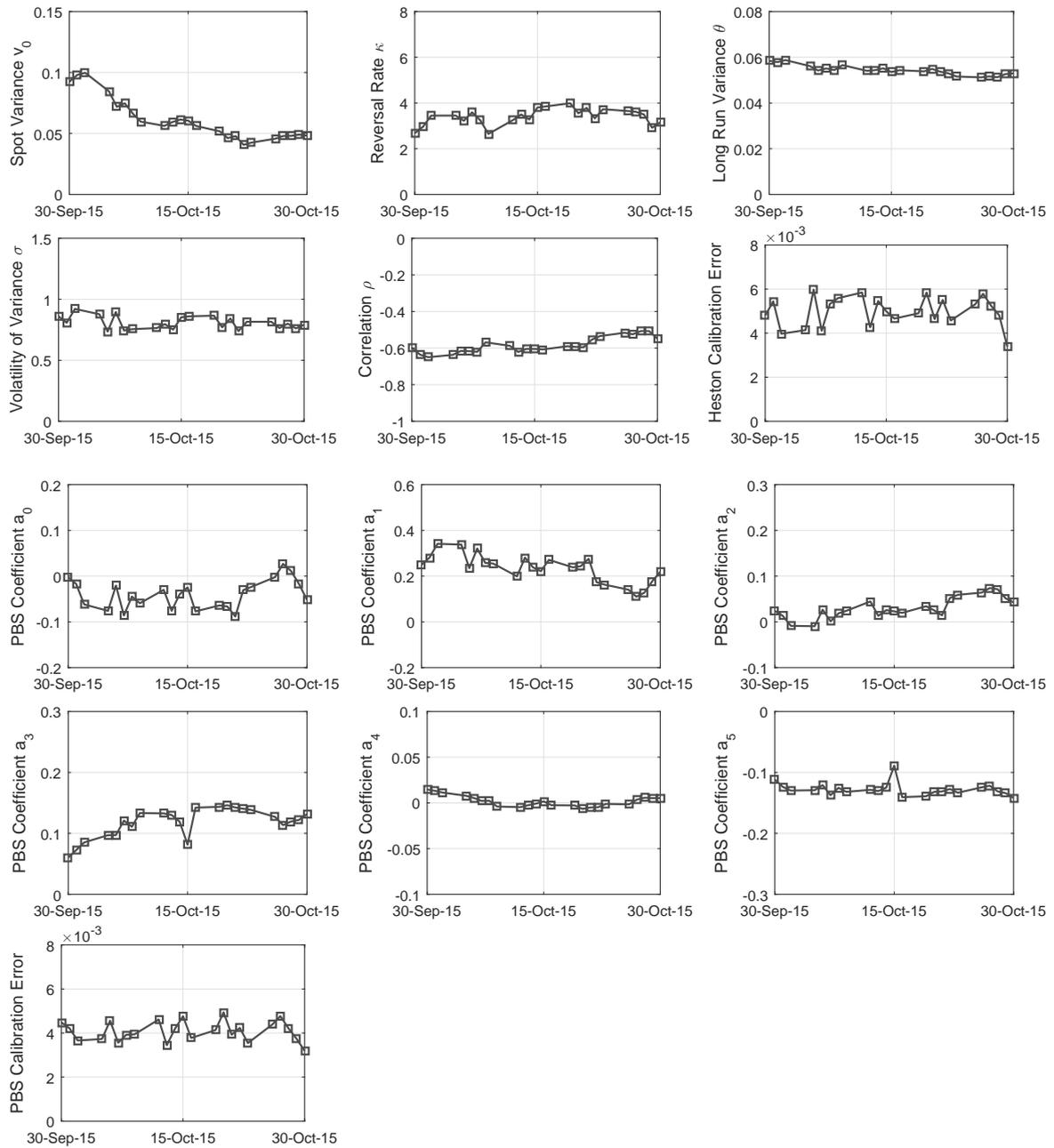


Exhibit 7: Model parameters and the IVRMSE in-sample fit Heston and PBS obtained from the sequential calibration to DAX index options.

Pricing Results

Now, we present the results of our repricing study outlined in the previous section. In particular, we compare the reported gross margins with the model-based gross margins on each certificate's first trading day.

Discount Certificates

Exhibit 8 shows summary statistics. Our results seem to be considerably robust to the particular choice of the option pricing model as Heston, PBS and Nadaraya-Watson imply similar results for the European-style discounters. Overall, in terms of the mean absolute error (MAE) and the root mean squared error (RMSE), the differences between reported and model-based are relatively low across issuers, with only Vontobel as an exception. In fact, by just considering averages, we have mean reported gross margins of 0.40% p.a. compared to 0.53% p.a. from Heston, 0.46% p.a. from PBS and 0.54% p.a. from the Nadaraya-Watson smoother. It is noticeable, that based on model prices, some banks even seem to offer structured products with negative gross margins – however this might be caused by small differences in the applied pricing methods and on the other hand by their respective trading books being axed at certain strikes, i.e. traders are especially willing to buy options at these levels to hedge other transactions.

Considering individual issuers, we find that although Vontobel reports the highest gross margins on average, our model-based results are 1% lower and well in the range of those from the market competitors. This is also reflected in MAE and RMSE which is twice as high as for other issuers. A potential explanation for these findings can be the fact that Vontobel's own treasury is funding these certificates higher than by our CDS approximation (see the preceding section). Whereas Citigroup, Goldman Sachs and HSBC show margins on average roughly in line with those we have calculated, BNP Paribas, Commerzbank, Deutsche Bank, and DZ Bank state explicitly lower average margins compared to results based on the Heston, PBS and the Nadaraya-Watson approach. However, with the exception of BNP Paribas (which contributes only 17 products to our total sample of 501), these deviations are rather small.

In total, our results from all three pricing approaches indicate that the average gross issuer margins are smaller than 1% which is in line with the results of Baule [2011] and Döhrer et al. [2013].

Capped Bonus Certificates

For the subsample of capped bonus certificates, Exhibit 9 presents the summary statistics of our repricing results compared to the annualized gross margins reported by each issuer.

For gross margins based on the Heston model, the differences are in a reasonable range and may fall under some pricing and calibration uncertainty. This is also reflected by MAE and RMSE being equally low across all issuers. In particular, the reported gross margin is with 1.10% p.a. very close to 1.35% p.a., the average based on the Heston model. Similar to the discounter study, BNP Paribas' structured products contain the largest average margin with 2.67% p.a. which is, however, broadly inline with a reported gross mean of 2.73%. The average margin of the capped bonus certificates issued by Deutsche and DZ Bank lie with level of 0.84% p.a. and 0.62% p.a. clearly above the reported values (0.28% and 0.19%). Similarly, Goldman Sachs exhibits an average margin of 1.44% p.a. versus an reported mean level of 1.28% p.a. while for the capped bonus certificates of Vontobel we calculated 1.60% p.a. versus a reported mean of 1.47%.

Not surprisingly, we can see considerably negative gross margins under the PBS and Nadaraya-Watson setup. This can be explained by the inability of the Gaussian Black-Scholes model to price the embedded barrier option of a capped bonus certificates adequately. With its simple assumptions of constant volatilities in time and normally distributed log-returns, even the generalized Black-Scholes setup is especially not able to capture the heavy tail behavior of the underlying and therefore overrates the value of the path-dependent down-and-out put. As it is well known by academics and practitioners, the stochastic volatility of the Heston model is a more reasonable choice for pricing such barrier options.

Issuer	# Products		Gross margin in % p.a.				Deviation from reported			
			Mean	Std.	Min.	Max.	MAE	RMSE	Min.	Max.
BNP Paribas	17	Reported	1.41	0.10	1.34	1.66				
		Heston	2.10	0.08	2.00	2.26	0.69	0.70	-0.78	-0.58
		PBS	2.09	0.12	1.93	2.33	0.68	0.68	-0.80	-0.57
		NW	2.27	0.13	2.05	2.52	0.86	0.86	-1.01	-0.64
Citigroup	73	Reported	0.68	0.12	0.31	0.82				
		Heston	0.64	0.57	0.03	1.92	0.51	0.67	-1.53	0.70
		PBS	0.66	0.59	0.08	2.05	0.51	0.68	-1.56	0.69
		NW	0.68	0.58	0.09	2.04	0.49	0.67	-1.53	0.69
Commerzbank	144	Reported	0.05	0.01	0.04	0.08				
		Heston	0.43	0.55	-0.12	1.39	0.49	0.67	-1.35	0.17
		PBS	0.39	0.55	-0.12	1.46	0.43	0.65	-1.41	0.18
		NW	0.47	0.54	-0.18	1.35	0.52	0.68	-1.31	0.23
Deutsche Bank	45	Reported	0.11	0.23	0.02	0.81				
		Heston	0.40	0.30	-0.07	1.04	0.31	0.37	-0.60	0.09
		PBS	0.32	0.36	-0.15	1.10	0.30	0.36	-0.67	0.18
		NW	0.36	0.25	0.01	0.94	0.26	0.31	-0.49	0.01
DZ Bank	62	Reported	0.01	0.00	0.00	0.02				
		Heston	0.58	0.16	0.27	0.92	0.57	0.59	-0.92	-0.25
		PBS	0.45	0.23	-0.12	0.94	0.45	0.50	-0.93	0.12
		NW	0.62	0.20	0.22	0.99	0.61	0.64	-0.98	-0.22
Goldman Sachs	20	Reported	0.73	0.03	0.69	0.77				
		Heston	0.39	0.07	0.27	0.57	0.35	0.36	0.17	0.44
		PBS	0.43	0.08	0.34	0.67	0.30	0.31	0.08	0.41
		NW	0.41	0.10	0.26	0.62	0.32	0.33	0.13	0.45
HSBC	85	Reported	0.18	0.04	0.04	0.27				
		Heston	0.35	0.13	0.00	0.66	0.17	0.20	-0.47	0.04
		PBS	0.25	0.14	-0.09	0.60	0.10	0.13	-0.42	0.19
		NW	0.37	0.12	-0.04	0.65	0.20	0.22	-0.40	0.20
Vontobel	55	Reported	1.58	0.31	1.38	2.20				
		Heston	0.57	0.55	-0.33	1.55	1.01	1.10	0.41	1.89
		PBS	0.37	0.55	-0.33	1.48	1.21	1.27	0.57	1.86
		NW	0.42	0.55	-0.54	1.75	1.16	1.24	0.44	2.10
<i>Overall</i>	501	Reported	0.40	0.54	0.00	2.20				
		Heston	0.53	0.53	-0.33	2.26	0.49	0.64	-1.53	1.89
		PBS	0.46	0.54	-0.32	2.34	0.47	0.66	-1.56	1.86
		NW	0.54	0.54	-0.55	2.52	0.52	0.68	-1.53	2.10

Exhibit 8: Summary statistics of reported and model-based annualized gross margins for discount certificates. Deviation is defined as reported minus model-based margin. MAE and RSME stands for mean absolute error and root mean squared error, respectively.

Issuer	# Products		Gross margin in % p.a.				Deviation from reported			
			Mean	Std.	Min.	Max.	MAE	RMSE	Min.	Max.
BNP Paribas	58	Reported	2.73	0.52	2.29	3.51				
		Heston	2.67	0.35	2.10	3.25	0.22	0.24	-0.44	0.43
		PBS	-0.30	1.26	-3.46	2.10	3.03	3.39	0.19	6.93
		NW	-0.31	1.27	-3.62	2.16	3.04	3.42	0.13	7.09
Deutsche Bank	49	Reported	0.42	0.20	0.02	0.68				
		Heston	0.84	0.16	0.32	1.12	0.42	0.47	-0.90	-0.21
		PBS	-2.44	2.31	-9.03	0.28	2.86	3.72	-0.02	9.60
		NW	-2.48	2.40	-9.12	0.44	2.92	3.82	-0.18	9.69
DZ Bank	77	Reported	0.20	0.01	0.20	0.22				
		Heston	0.62	0.21	0.12	1.06	0.42	0.46	-0.86	0.08
		PBS	-2.27	1.84	-8.89	0.23	2.47	3.08	-0.03	9.10
		NW	-2.28	1.97	-9.42	0.23	2.48	3.16	-0.02	9.63
Goldman Sachs	18	Reported	1.28	0.02	1.27	1.31				
		Heston	1.44	0.30	0.80	1.99	0.25	0.34	-0.72	0.51
		PBS	-1.44	1.71	-5.22	0.61	2.73	3.20	0.66	6.51
		NW	-1.52	1.75	-5.02	0.65	2.80	3.28	0.62	6.32
Vontobel	11	Reported	1.47	0.08	1.39	1.57				
		Heston	1.60	0.78	-0.43	2.57	0.53	0.78	-1.17	2.00
		PBS	-1.92	2.13	-6.23	0.76	3.38	3.97	0.63	7.78
		NW	-1.68	1.72	-4.32	0.98	3.15	3.57	0.41	5.87
<i>Overall</i>	501	Reported	1.10	1.10	0.02	3.50				
		Heston	1.35	0.91	-0.42	3.25	0.36	0.43	1.17	2.00
		PBS	-1.68	2.02	-9.03	2.10	2.78	3.38	-0.03	9.60
		NW	-1.70	2.07	-9.42	2.17	2.17	2.80	-0.18	9.69

Exhibit 9: Summary statistics of reported and model-based gross annualized margins for capped bonus certificates. Deviation is defined as reported minus model-based margin. MAE and RMSE stands for mean absolute error and root mean squared error, respectively.

Conclusion

Retail structured products feature individual payoff profiles, reduced market barriers and no significant transaction costs, which make them attractive for many investors. However, as short selling of these products is not possible and the issuing institutions also act as market makers, issuers can potentially realize high margins. To improve transparency on the German market, the Deutscher Derivate Verband and issuing banks decided in May 2014 to provide the issuer estimated value, a fair value of the certificates that should reflect the market price of the product among professionals. By publishing the issuer estimated value, banks make a statement on their gross margin, a value that contains the issuer's profit but also distribution and selling costs.

In the present paper, we provide a first look at these implicitly reported gross margins and assess their adequacy on a test sample of 501 discount and 213 capped bonus certificates on the DAX index. We have two main findings: Firstly, we deduce that reported gross margins vary considerably across issuers even on standard retail products such as the discount and capped bonus certificates. Secondly, our results confirm earlier academic findings on the size of the gross margins supposedly included in the issuer's prices and thus indicate that the average gross margins disclosed by the issuers can be explicitly verified. Model-based gross margins are quite close to those reported by the issuers given the various different sources of pricing and calibration uncertainty.

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Appendix

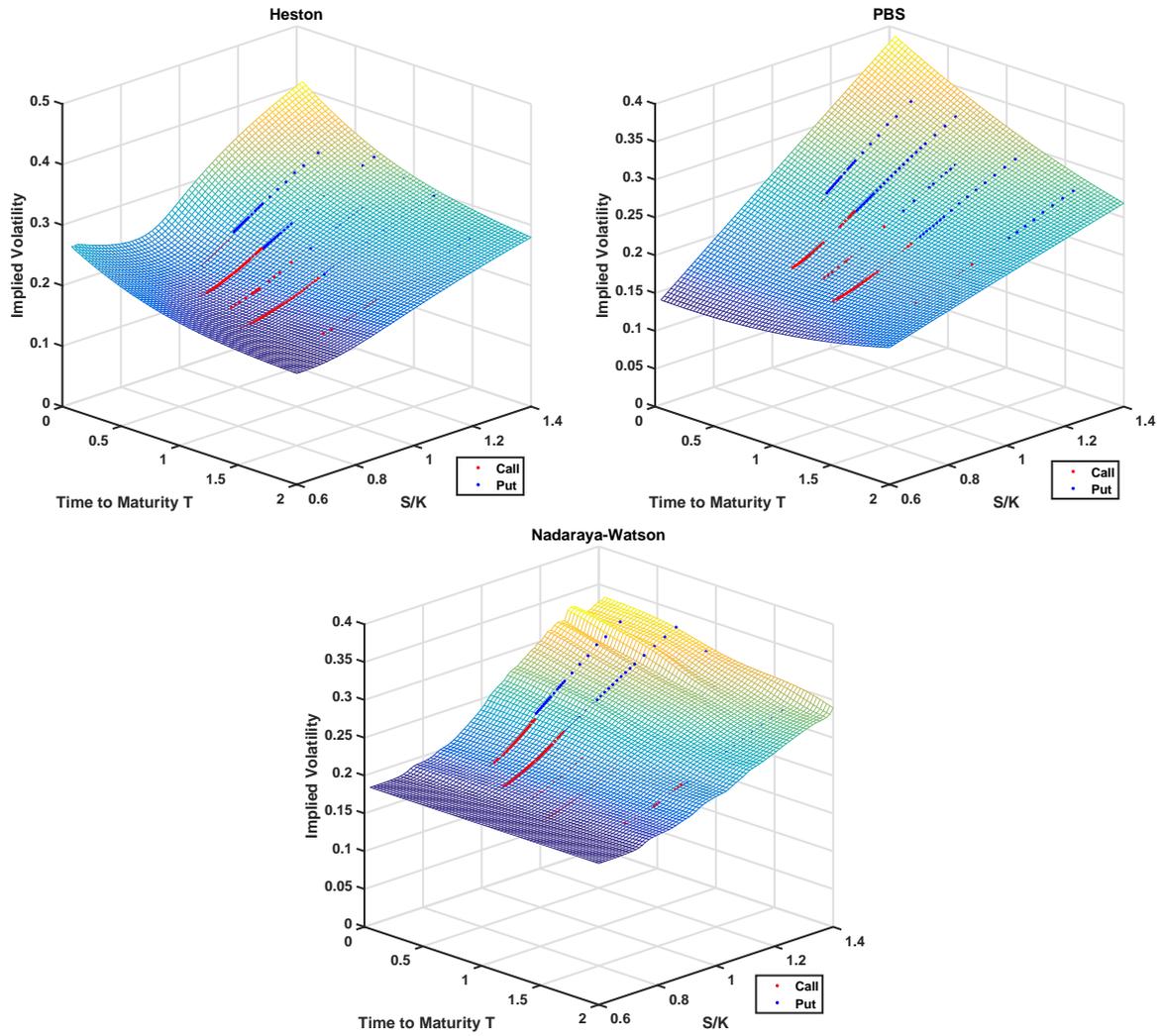


Exhibit 10: Smoothed implied volatility surfaces from Heston, PBS and Nadaraya-Watson on September 30th, 2015.

