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IMPLIED CORRELATION INDICES AND VOLATILITY FORECASTING

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ABSTRACT

Implied volatility indices are an important measure for 'market fear' and well-known in academia and practice. Correlation is still payed less attention even though the CBOE started to calculate implied correlation indices for the S&P500 in 2009. However the literature especially on cross-country dependencies and applications is still quite thin. We are closing this gap by constructing an implied correlation index for the DAX and taking a deeper look at the (intercontinental) relationship between equity, volatility and correlation indices. Additionally, we show that implied correlation could improve implied volatility forecasting.

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JEL SUBJECT CLASSIFICATION

C10, C53, G15, G10

1 Motivation

For foreign exchange options, the concept of implied correlation is well-known and easy to grasp as cross-rates help to uniquely identify implied correlation by just considering classical vanilla options. Consequentially, several studies are available in this area (e.g. Siegel (1997), Campa and Chang (1998) or Walter and Lopez (2000)) showing among other things a positive dependence structure between volatility and correlation.

However, for equity indices, the situation is more delicate as one would need reliable options prices of bivariate options on all index components. To overcome this obstacle, the assumption of equal implied correlation for all index constituents (*equicorrelation*) is usually made. In 2009, as the first global exchange, the CBOE began calculating implied correlation indices for the S&P500 (also called 'SPX') as a measure for an average expected market dependence. The heuristic of their methodology consists of comparing an estimator for the at-the-money (ATM) implied volatility of the index with a weighted sum built from ATM volatilities of a 50-stock-subindex (cf. CBOE (2009)) and is implicitly using the *equicorrelation* assumption.

While several studies like Fleming et al. (1995), Christensen and Prabhala (1998) or Fernandes et al. (2014) already stressed the importance of implied volatility indices like CBOE's VIX, the literature on implied equity correlation is still quite thin. One of the first authors to actually calculate such an index were Skintzi and Refenes (2005) who showed that it provides a better forecast for future correlation than using historical data. Furthermore, Driessen et al. (2009) deducted that implied correlation might carry a negative risk premium while Härdle and Silyakova (2012) have investigated correlation based trading strategies.

Our aim in the present study is to further stress the importance of the implied correlation concept. In order to do that, we calculate an implied correlation index for the German DAX and investigate the intercontinental dependencies with the SPX and its volatility and correlation indices. Even though, for the sake of comparability, we will mirror the procedure from CBOE, we want to remark that Linders and Schoutens (2014) recently pointed out some potential weaknesses of this concept. Furthermore, we will show that the inclusion of implied correlation in a classical ARMAX

model leads to an improved forecasting of implied volatility levels. This could have important consequences for market makers and portfolio managers as implied volatility is a significant pricing factor for derivatives.

All computations were done in MATLAB via the MFE toolbox (cf. Sheppard (2013)).

2 Correlation indices

We will construct DAX-implied correlation indices based on the procedure of the CBOE S&P 500 Implied Correlation Index (called 'CIX' in this study) described in detail by CBOE (2009). The basic idea is to compare for each day $t \geq 0$, the implied volatility $\sigma_{\text{Index},t}$ of an ATM-index option with the respective implied volatilities σ_{it} of (generally) each individual component. Under the *equicorrelation* assumption we can derive the (average) implied correlation of the index and define

$$\text{Implied Correlation Index}_t = 100 \cdot \frac{\sigma^2_{\text{Index},t} - \sum_{i=1}^{N} w_{it}^2 \sigma_{it}^2}{2 \sum_{i=1}^{N-1} \sum_{j>i}^{N} w_{it} w_{jt} \sigma_{it} \sigma_{jt}} \tag{1}$$

where $(w_{it})_{i=1,\ldots N}$ are the components' index weights given by

$$w_{it} = \frac{S_{it} \cdot X_{it}}{\sum_{i=1}^{N} S_{it} \cdot X_{it}}, \quad i = 1, \ldots N, \tag{2}$$

with $(S_{it})_{i=1,\ldots N}$ being the share prices and $(X_{it})_{i=1,\ldots N}$ the number of free-floating shares.

Exact ATM-volatilities for both, index and constituents, were approximated as outlined in CBOE (2009). Similar to the calculation of the VDAX-NEW (cf. Deutsche Börse (2007)), we interpolated the riskless term structure by taking EONIA, 3m-, 6m-, 9m- and 12m-EURIBOR in addition to the return of the 2-year REX representing syntectic prices for German government bonds.

Our chosen time frame starts after the triple witching day 20th December 2010 and goes until 16th November 2012, the expiration day of the respective CIX 2012, leading to three DAX correlation indices using options maturing on 16th December 2011, 15th December 2012 and 20th December 2013 (cf. Table 1). As a consequence, for each trading day, 2 correlation indices are available (inline with the CIX).

Correlation indices	Time period
CDAX 2011	20.12.2010 – 12.12.2011
CDAX 2012	01.12.2011 – 16.11.2012
CDAX 2013	01.12.2011 – 16.11.2012

Table 1: DAX correlation indices

All necessary data series were taken from *Thomson Datastream*. Due to incomplete option data for Commerzbank AG, Fresenius SE & Co. KGaA, Merck KGaA and MAN SE these underlyings were excluded from the correlation indices. In contrast to the CIX which are only based on the 50 top components in terms of market cap, the CDAX indices are principally built on the whole DAX. We want to remark that on 24st September 2012 MAN SE and Metro Group were excluded from the index and exchanged for Continental AG and Lanxess AG. To avoid a potential bias by increasing the considered single stocks, we interchanged Metro Group with Continental AG and left Lanxess AG out. In terms of time horizon and number of stocks, our data setup is therefore similar to Härdle and Silyakova (2012).

The new CDAX indices, the VDAX-NEW (for ease of notation from now on just called 'VDAX'), the DAX and their respective US counterparts are depicted in Figure 1. The later ones are publicly available data on CBOE's website. To allow a longer term analysis, we additionally computed a rolling correlation index by taking the mean of both available indices each day and rolling the CDAX 2011 into its 2013 counterpart over 10 days before expiry.

As can be seen, even though the absolute levels are similar over time, the CDAX is more volatile than its US counterpart. This can be explained by the fact that only 26 index components are used while the CIX is calculated on the basis of 50 single stocks.

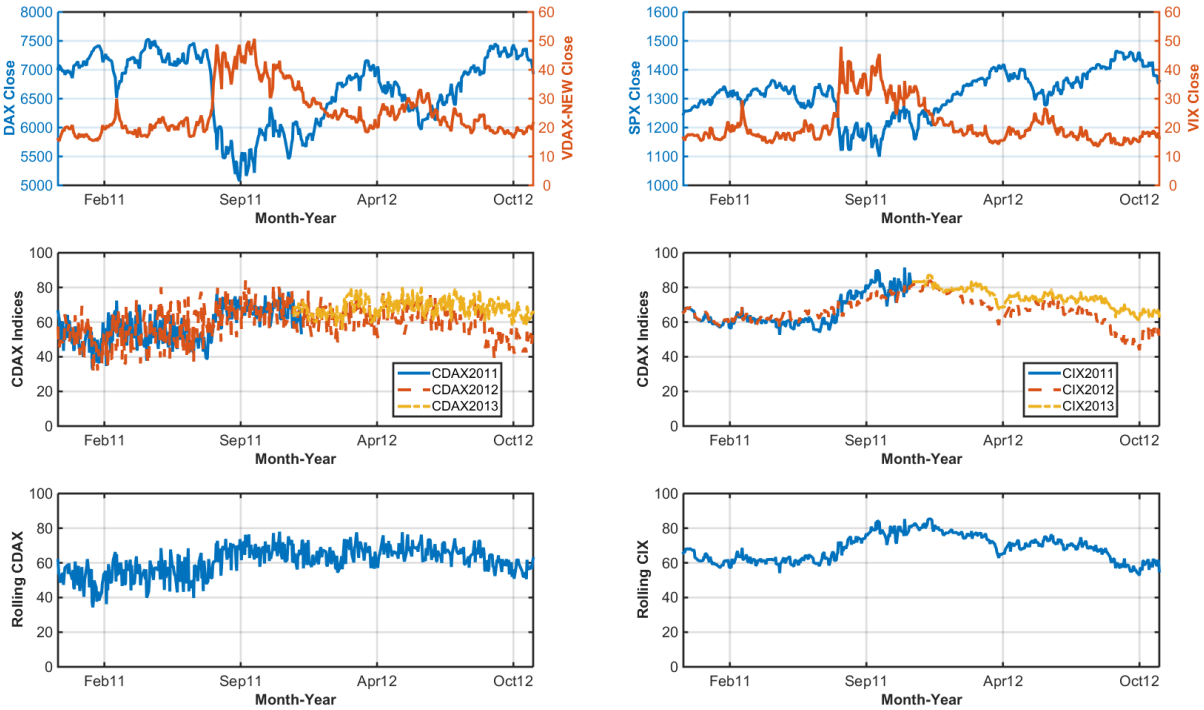

Figure 1: Index time series.

3 Dependence Analysis

In the literature, it is well known, that equity and volatility indices exhibit a negative correlation and a quick look at Figure 1 confirms this theory for our chosen data set as well. When considering implied correlation, the picture is not that clear even though e.g. the financial crisis 2007/2008 has led to increasing correlation at least on the downside (cf. Mittnik (2014) for a good overview on supporting studies). To get a clearer picture of the pure dependence structure in our data we applied a univariate ARMA-GARCH filtering to the log returns of each index.

Data	Type	Ljung-Box	ARCH-LM	Error distribution
DAX	ARMA(0,0)-GARCH(1,1)	0.4137	0.8830	Hansen's skewed-t
VDAX	ARMA(0,1)-GARCH(1,0)	0.1836	0.5414	Hansen's skewed-t
CDAX	ARMA(1,1)-GARCH(1,1)	0.6006	0.2910	Hansen's skewed-t
SPX	ARMA(1,1)-GARCH(1,0)	0.1172	0.4681	Hansen's skewed-t
VIX	ARMA(1,1)-GARCH(1,1)	0.2866	0.5047	Hansen's skewed-t
CIX	ARMA(1,1)-GARCH(1,1)	0.9269	0.6771	Hansen's skewed-t

Table 2: Final time series models.

We estimated all setups from ARMA(0,0)-GARCH(0,0) to ARMA(1,1)-GARCH(1,1) using normal, Student-t and Hansen's skewed-t distributed errors. Afterwards we dropped all models for which the Ljung-Box and/or ARCH-LM test could not reject the null hypothesis and selected for each time series the best of the remaining in terms of BIC. The results can be found in Table 2.

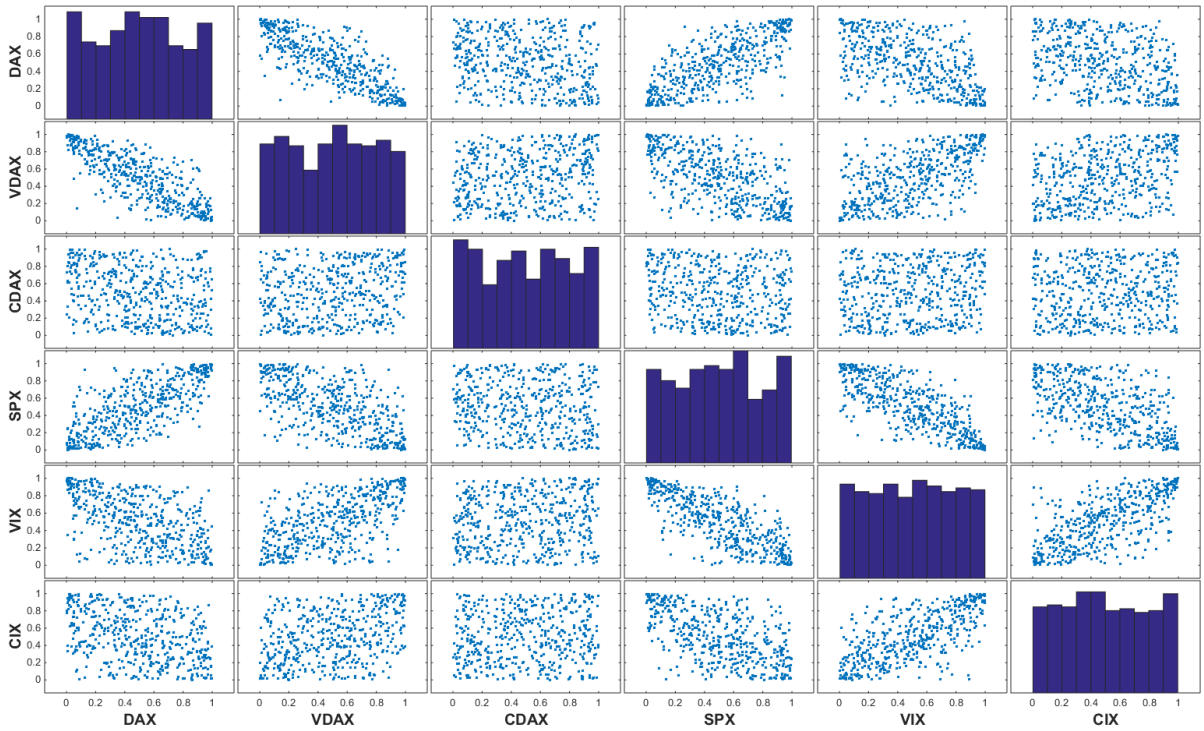

Figure 2: Filtered and transformed time series.

Using the estimated parameters of the selected error distributions, we transformed the residuals to univariate uniform distributions such that only the pure dependencies remain. The bivariate scatter plots are shown in Figure 2.

For the equity and volatility indices within each country we can clearly see the negative tail correlation. Also, DAX and SPX as well as VDAX and VIX show a positive tail dependence, even though it looks not as strong. When it comes to implied correlation the picture gets more ambivalent: while the CIX and SPX show negative tail dependence, it gets positive when looking at CIX and VIX confirming the industry-wide paradigm that in times of crisis, correlation goes up similar to volatility. However, the CDAX does not provide a clear picture even though one can still calculate a Pearson-correlation of -0.24 with the DAX and $+0.21$ with the VDAX residuals. The weaker dependence compared to the CIX could be explained by the DAX-implied correlation index showing a greater variability due to the fact that it is based on just about half the amount of single stocks.

4 VDAX forecasting based on implied correlation

Even though the study of Zhou (2013) indicates that implied correlation could hold some predictive power about its related equity index (and its realized volatility) this can only work to some extend if one believes in the efficient market hypothesis. However, as we will show in this section, it might be especially interesting for practitioners to use correlation indices to forecast implied volatility, a main pricing factor for derivatives. Therefore we model the logarithmic VDAX returns $(r_t^{\mathrm{VDAX}})_{t\in\mathbb{Z}}$ by an ARMAX setup via

$$r_t^{\mathrm{VDAX}} = a_0 + a_1 r_{t-1}^{\mathrm{VDAX}} + \gamma_1 r_{t-1}^{\mathrm{CDAX}} + \gamma_2 r_{t-1}^{\mathrm{VIX}} + \gamma_3 r_{t-1}^{\mathrm{CIX}} + \varepsilon_t + b_1 \varepsilon_{t-1} \tag{3}$$

for $t \in \mathbb{Z}$ with $(\varepsilon_t)_{t\in\mathbb{Z}} \overset{iid}{\sim} (0,1)$ and estimate the model under all possible parameter restriction based on the first

80% of our data. Afterwards the remaining 20% are used to evaluate the out of sample one-step forecasts, keeping the parameters fixed, via their mean-squared-error (MSE) defined by

$$\text{MSE} = \frac{1}{n-m} \sum_{t=m+1}^{n} [\widehat{r_t^{\text{VDAX}}} - r_t^{\text{VDAX}}]^2 \tag{4}$$

where n denotes the length of the whole time series while m is given by the training cutoff (80% in our case).

Of the 32 considered setups, the top performers in terms of the MSE are listed in Table 3. In fact, the best presented model has also the lowest BIC. In general, we can see that the CDAX, VIX and CIX have a positive impact on the VDAX, confirming our findings from the previous section. The inclusion of implied correlation indices improves volatility forecasting to a level which we were not able to reach with classical ARMA models even when allowing higher orders. Interestingly, one can see that although the top two performers do not include lagged VDAX returns, we are not able to find any significant autocorrelations or ARCH-effects in the error terms anymore.

ARMA	Exogenous returns	BIC	Ljung-Box	ARCH-LM	α_0	α_1	β_1	γ_1	γ_2	γ_3	MSE
(0,0)	CDAX,VIX,CIX	-1043	0.3672	0.4705	0.0009	–	–	0.0209	0.1618	0.1202	0.002445
(0,0)	VIX,CIX	-1042	0.3489	0.4520	0.0009	–	–	–	0.1684	0.1064	0.002449
(1,0)	CIX	-1028	0.3069	0.3114	0.0009	0.0652	–	–	–	0.3380	0.002450
(1,0)	CDAX,CIX	-1029	0.3494	0.3205	0.0009	0.0531	–	0.0251	–	0.3487	0.002451
(0,0)	CDAX,VIX	-1042	0.3702	0.4627	0.0009	–	–	0.0185	0.1946	–	0.002455
(1,0)	–	-1018	0.2867	0.3110	0.0009	0.1118	–	–	–	–	0.002736
(0,1)	–	-1018	0.3350	0.3112	0.0010	–	0.1222	–	–	–	0.002819

Table 3: ARMAX-models and out-of-sample prediction error measured by MSE.

5 Concluding remarks

We calculated an implied correlation index for the German DAX and analyzed the dependence structure with its equity and volatility relatives as well as with the corresponding SPX related indices. Furthermore we have shown that the inclusion of implied correlation can effectively improve implied volatility forecasting which suggests possible applications especially in derivative pricing.

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